1. Find the derivative of $g(x) = \frac{xe^{x^2+1}}{3x+2}$. Do not simplify your answer.

Solution: Applying the quotient and product rules, we obtain

$$g'(x) = \frac{(3x+2)\frac{d}{dx}\left[xe^{x^2+1}\right] - xe^{x^2+1}\frac{d}{dx}[3x+2]}{(3x+2)^2}$$
$$= \frac{(3x+2)\left(xe^{x^2+1}(2x) + e^{x^2+1}\right) - xe^{x^2+1}(3)}{(3x+2)^2}.$$

2. If $f(9) = 1$, $f'(9) = 3$, $g(1) = 4$, $g'(1) = -2$, and $h(x) = x^{3/2}g(f(x))$, then what is $h'(9)$? Simplify your answer.

Solution: Via the product and chain rules, we obtain $h'(x) = x^{3/2}g'(f(x))f'(x) + g(f(x))(3/2)x^{1/2}$. Thus

$$\begin{aligned} h'(9) &= 9^{3/2}g'(f(9))f'(9) + g(f(9))(3/2)9^{1/2} \\ &= 27g'(1)(3) + g(1)(3/2)(3) \\ &= 27(-2)(3) + 4(3/2)(3) \\ &= -144. \end{aligned}$$

3. Suppose $s(t) = t^3 - 9t^2 + 15t$ is the position of a particle traveling along a coordinate line, where s is measured in meters and t, in seconds. At what times will the particle be at rest?

Solution: The particle will be at rest when its velocity is zero; that is, when the rate of change in the particle's position with respect to time is 0. Thus we solve $\frac{ds}{dt} = 0$:

$$3t^2 - 18t + 15 = 0; \quad \text{equivalently} \quad 3(t-1)(t-5) = 0.$$

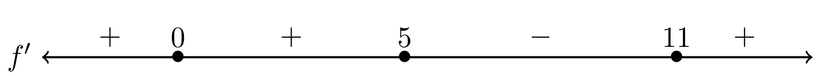
Hence, the particle is at rest when $t = 1$ and when $t = 5$.

4. Find and classify the critical numbers of $f(x) = x^5(x-11)^6$, indicating for each critical number whether it yields a relative maximum value of f, a relative minimum value, or neither.

Solution: We have

$$\begin{aligned} f'(x) &= x^5 \cdot 6(x-11)^5 + (x-11)^6 \cdot 5x^4 \quad \text{(by the Product Rule)} \\ &= x^4(x-11)^5(6x + 5(x-11)) \\ &= x^4(x-11)^5(11x-55) \\ &= 11x^4(x-11)^5(x-5). \end{aligned}$$

We see that the critical numbers of f are $0, 5$, and 11, each of these being a zero of f'.



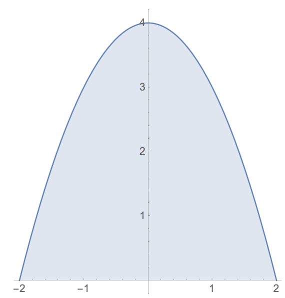
The f' sign-line above shows that f is increasing on the intervals $(-\infty, 0)$, $(0, 5)$, and $(11, \infty)$, while f is decreasing on $(5, 11)$. Because f is continuous, we conclude via the first-derivative test that 0 yields neither a relative maximum nor a relative minimum of f, that 5 yields a relative maximum, and that 11 yields a relative minimum.

5. Find $f(x)$ given that $f'(x) = 3x^2 + 4x - 1$ and $f(2) = 9$.

   Solution: Anitdifferentiating both sides of the equation $f'(x) = 3x^2 + 4x - 1$, we find $f(x) = x^3 + 2x^2 - x + C$ for some constant C. Now, using $f(2) = 9$, we obtain $9 = 14 + C$, so that $C = -5$. Thus $f(x) = x^3 + 2x^2 - x - 5$.

6. Find the area of the region bounded by the curves $y = 4 - x^2$ and $y = 0$.

   Solution: The region whose area is to be computed is shaded in blue below.



The definite integral $\int_{-2}^{2}(4 - x^2)\,dx$ yields the area:

$$\int_{-2}^{2}(4 - x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = (8 - 8/3) - (-8 + 8/3) = \frac{32}{3}.$$

7. The number of items produced by a manufacturer is given by

$$p = 100xy^3,$$

where x is the amount of capital and y is the amount of labor, amounts that change over time. At a particular point in time:

(i) the manufacturer has 2 units of capital;
(ii) capital is increasing at a rate of 1 unit per month;
(iii) the manufacturer has 3 units of labor; and
(iv) labor is decreasing at a rate of 1/3 unit per month.

Determine the rate of change in the number of items produced at this point in time.

Solution: Differentiating with respect to time we have

$$\frac{dp}{dt} = 100\left(x \cdot 3y^2\frac{dy}{dt} + y^3\frac{dx}{dt}\right)$$

At the moment when $x = 2$, $\frac{dx}{dt} = 1$, $y = 3$ and $\frac{dy}{dt} = -1/3$, we have that

$$\frac{dp}{dt} = 100\left(2 \cdot 3 \cdot 3^2 \cdot \left(\frac{-1}{3}\right) + 3^3 \cdot 1\right) = 900 \text{ items/month.}$$

8. Find $\int (e^x + 1)^2\, dx$.

Solution:

$$\begin{aligned}
\int (e^x + 1)^2\, dx &= \int (e^{2x} + 2e^x + 1)\, dx \\
&= \int e^{2x}\, dx + 2\int e^x\, dx + \int 1\, dx \\
&= \frac{1}{2}\int e^u\, du + 2e^x + x + C \qquad (u = 2x; \frac{1}{2}du = dx) \\
&= \frac{1}{2}e^u + 2e^x + x + C \\
&= \frac{1}{2}e^{2x} + 2e^x + x + C
\end{aligned}$$

Grade the problem "correct" even if you forgot to include +C .

9. Evaluate $\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}\, dx$.

Solutions: We have

$$\begin{aligned}
\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}\, dx &= \int_e^{e^4} (\ln(x))^{-1/2}x^{-1}\, dx \\
&= \int_1^4 u^{-1/2}\, du \qquad (u = \ln x, du = x^{-1}\, dx) \\
&= \left[2u^{1/2}\right]_1^4 \\
&= 2(4)^{1/2} - 2(1)^{1/2} \\
&= 2.
\end{aligned}$$

10. Over the time interval $1 \leq t \leq 100$ the temperature of a freezer compartment is given by

$$f(t) = \frac{t-4}{t^2},$$

where t is measured in hours and $f(t)$ is measured in degrees Celsius. What's the maximum temperature of the freezer over this time interval?

Solution: Because f is continuous on $[1, 100]$ (it's a rational function whose domain includes the entire interval $[1, 100]$), the Extreme-Value Theorem tell us that f attains a maximum value at some number, say c, in $[1, 100]$. This number c must either be an endpoint of $[1, 100]$ or a critical number of f in $(1, 100)$. We have

$$f'(t) = \frac{t^2(1) - (t-4)2t}{t^4} = \frac{8-t}{t^3},$$

and thus 8 is the only critical number of f inside $(1, 100)$. The largest of the three numbers $f(1), f(8), f(100)$ will be the maximum value of f on $[1, 100]$. Since $f(1) = -3$, $f(8) = 1/16$, and $f(100) = 96/10000 < 100/10000 = 1/100$, we see the maximum temperature of the freezer over the time interval $[1, 100]$ is $1/16$ degrees Celsius.