Making Sense of Probability as a Set of Measures

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Abstract

Is there a concept of probability that meets all the needs of physical science? If so, how do we know this?

We attempt to understand a set $\mathcal{M} = \{\mu\}$ of probability measures, although not as a traditional compound statistical hypothesis in which one of the measures in $\mathcal{M}$ is a true description. Our interest is rather in the case that none of the individual measures in $\mathcal{M}$ provides an adequate description of the potential behavior of the physical source as actualized in the form of a long, finite time series. It is to be the whole set $\mathcal{M}$ that describes the potential behavior, and this distinction has operational significance in terms of the time series data that is anticipated from the physical source. While we adopt the same mathematical model that is central to the theory of subjective coherent imprecise probability (e.g., see Walley [1991]), we endow this model with an objective, frequentist interpretation in place of a behavioral subjective one, and bend it to a different purpose.

We seek to model sources that have highly irregular behavior and not to model states of belief or knowledge that are assuredly imprecise. Support for the existence of such chaotic sources is lent by the following quotation from Kolmogorov [1983],

"In everyday language we call random those phenomena where we cannot find a regularity allowing us to predict precisely their results. Generally speaking, there is no ground to believe that random phenomena should possess any definite probability."

Our goal is to provide a consistent methodology that can both simulate a time series realization given a model $\mathcal{M}$ and estimate the model from time series data.