Analysis general exam syllabus

Revised March 2023

Basics (common for complex and real parts)

1. Elementary set operations, countable and uncountable sets.
2. Open, closed, compact, and connected sets on the line and in Euclidean space.
3. Completeness, infima and suprema, limit points, $\liminf$, $\limsup$.
4. Bolzano-Weierstrass, Heine-Borel theorems.
5. Continuous, uniformly continuous, differentiable functions.
6. Extreme value, intermediate value, mean value theorems.

Complex analysis

1. Series of functions, power series, and power series of elementary functions, uniform convergence, Weierstrass M test. Formula for the radius of convergence of a power series.
2. Analytic and harmonic functions, Cauchy-Riemann equations.
3. Power series and Laurent series.
4. Elementary conformal mappings, fractional linear mappings. The Cayley transform.
5. Cauchy's integral theorem and Cauchy integral formula. Morera's theorem. Goursat's theorem.
6. Power series representation is equivalent to complex differentiability, and the power series converges in any ball where the function is complex differentiable. Classification of singularities. Meromorphic functions. Casorati-Weierstrass (Sokhotski) theorem.
7. Argument principle, open mapping theorem, maximum principle, Rouché's theorem, Schwarz's lemma, Liouville's theorem. Hurwitz' theorem.
8. Cauchy's residue theorem, evaluation of definite integrals. Evaluation of infinite series via residue theory.
9. Normal families. Montel's theorem. Riemann mapping theorem.

Real analysis

1. σ-algebras of sets.
2. Lebesgue measures and abstract measures, signed measures. Lebesgue-Stieljtes measures on the real line and their correspondence with increasing, right continuous functions.
3. Measurable functions. Approximation by simple functions. Riemann and Lebesgue integrals.
4. Monotone convergence and dominated convergence theorems, Fatou's lemma.
5. Product spaces and product measure, Fubini-Tonelli theorems.
6. Absolute continuity of measures, Radon-Nikodym theorem. Lebesgue-Radon-Nikodym decomposition.
7. Hardy-Littlewood maximal function: the maximal inequality for the Hardy-Littlewood maximal function and the Vitali covering lemma. The Lebesgue differentiation theorem.
8. Absolute continuity of functions, differentiation and the Fundamental Theorem of Calculus for absolutely continuous functions. The correspondence between absolutely continuous functions on $\mathbb{R}$ and measures on $\mathbb{R}$ which are absolutely

continuous with respect to the Lebesgue measure. Bounded variation functions and their correspondence with complex measures on $\mathbb{R}$.

9. Hölder's inequality, Jensen's inequality.

10. L^p spaces, completeness. Approximation of L^p-functions on $\mathbb{R}^d$ by compactly supported continuous functions.

11. Hilbert space, projection theorem, Riesz representation theorem, orthonormal sets, L^2 spaces.

12. L^p-$L^{p'}$ duality when $\frac{1}{p} + \frac{1}{p'} = 1$.

13. Elementary Fourier series, Riesz-Fischer and Parseval theorems. Riemann-Lebesgue theorem. Dirichlet kernel.

14. Fourier transforms in $\mathbb{R}^d$, Plancherel and Parseval's theorems, Fourier transforms of derivatives and translations. Riemann-Lebesgue lemma. Hausdorff-Young's inequality.

15. Convolutions: Fourier transforms of convolutions, approximations to the identity, approximation of functions on $\mathbb{R}^d$ by compactly supported smooth functions. Young's inequality for convolution.

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