Syllabus for Analysis General Exam

Sets in Euclidean Space
Elementary set operations, countable and uncountable sets. Open, closed, compact, and connected sets on the line and in Euclidean space. Completeness, infima and suprema, limit points. lim inf, lim sup. Bolzano-Weierstrass, Heine-Borel theorems.

Functions
Continuous, uniformly continuous, differentiable functions. Extreme value, intermediate value, mean value theorems. Series of functions, power series, and power series of elementary functions, uniform convergence, Weierstrass *M* test.

Measure Theory
Lebesgue measures and abstract measures, signed measures, σ-algebras of sets. Measurable functions. Riemann and Lebesgue integrals. Monotone convergence and dominated convergence theorems, Fatou lemma.

Product spaces and product measure, Fubini's theorem. Absolute continuity, Radon-Nikodym theorem. Differentiation and the Fundamental Theorem of Calculus.

Linear Spaces
Hőlder's inequality, Jensen's inequality. L^p spaces, completeness. Hilbert space, projection theorem, Riesz representation theorem, orthonormal sets, L^2 spaces.

Elementary Fourier series, Riesz-Fischer and Parseval theorems. Riemann-Lebesgue theorem. Dirichlet kernel.

Analytic Functions
Analytic and harmonic functions, Cauchy-Riemann equations, power series, and Laurent series. Elementary conformal mappings, fractional linear mappings. Cauchy's theorem and Cauchy integral formula, Morera's theorem. Argument principle, open mapping theorem, maximum principle, Rouché's theorem, Schwarz's lemma, Liouville's theorem. Residue theorem, evaluation of definite integrals.

References
R. G. Bartle, *Elements of Real Analysis*; W. Rudin, *Principles of Mathematical Analysis*; H. L. Royden, *Real Analysis*; G. Folland, *Real Analysis*; L. V. Ahlfors, *Complex Analysis*; J. B. Conway, *Functions of a Complex Variable*; T. Gamelin, *Complex Analysis*.