

**Categorical Methods  
in Representation Theory and Quantum Topology  
University of Virginia, April 15–17**

**Main Schedule**

	Saturday morning <i>Maury 209</i> (Chair: Krushkal)	Sunday morning <i>Maury 209</i> (Chair: Wang)
	Beliakova (8:30–9:20am)	Negut (8:30–9:20am)
	Rose (9:30–10:20am)	Rozansky (9:30–10:20am)
	Coffee break (10:30–11:00am)	Coffee break (10:30–11:00am)
	Contributed talks (11:00am–12:30pm)	Im (11:00–11:50am)
Friday afternoon <i>Ridley G004</i> (Chair: Qi)	Saturday afternoon <i>Maury 209</i> (Chair: Chari)	
Chari (1:00–1:50pm)	Lunch break (12:30–2:00pm)	
Negrón (2:00–2:50pm)	Dhillon (2:00–2:50pm)	
Coffee break (3:00–3:30pm)	Coffee break (3:00–3:30pm)	
Sussan (3:30–4:20pm)	Contributed talks (3:30–5:00pm)	
Manion (4:30–5:30pm)		

**Contributed Talk Schedule**

Saturday Morning Session 1 <i>Maury 209</i> (Chair: Sussan)	Saturday Morning Session 2 <i>Maury 104</i> (Chair: Samperton)
McMillan (11:00–11:20am)	M. Zhang (11:00–11:20am)
Ebert (11:30–11:50am)	Gomez (11:30–11:50am)
Larson (12:00–12:20pm)	Akhmechet (12:00–12:20pm)
Saturday Afternoon Session 1 <i>Maury 209</i> (Chair: Im)	Saturday Afternoon Session 2 <i>Maury 104</i> (Chair: Akhmechet)
W. Zhang (3:30–3:50pm)	Samperton (3:30–3:50pm)
Bodish (4:00–4:20pm)	Li (4:00–4:20pm)
Shen (4:30–4:50pm)	Wilbert (4:30–4:50pm)

## Titles and abstracts of invited talks

**Anna Beliakova** Algebraization of low-dimensional Topology

*University of Zurich, Email: anna@math.uzh.ch*

*Abstract:* In my talk, I will give a complete algebraic description of the category 4HB of orientable 4-dimensional 2-handlebodies in terms of a certain braided Hopf algebra object  $H$ . A boundary functor from 4HB to the category 3Cob of connected 3-dimensional cobordisms is described algebraically by adding relations to  $H$ . I will use this approach to construct TQFT-functors on 4HB and 3Cob with values in a representation category of a (unimodular) ribbon Hopf algebra. Already the quantum  $\mathfrak{sl}(2)$  gives rise to interesting examples. I plan to finish by discussing relations with the famous problem in combinatorial group theory—the Andrews–Curtis conjecture. This is a joint work with Marco De Renzi based on results by Bobtcheva and Piergallini.

**Vyjayanthi Chari** Higher order Kirillov-Reshetikhin modules and monoidal categorification

*University of California, Riverside, Email: vyjayanthi.chari@ucr.edu*

*Abstract:* We discuss a generalization in type  $A_n$  of the well-known KR-modules for quantum affine algebras. We shall see that our generalization has many of the properties of the KR-modules. Moreover, they also allow us to classify all prime representation of the quantum affine algebra which are supported on only one node of the Dynkin diagram of  $A_n$ . We give a necessary and sufficient for a tensor product of such modules for a fixed node to be irreducible. Finally, we discuss how the analog of theory of monoidal categorification developed by Hernandez and Leclerc for KR-modules.

**Gurbir Dhillon** Kazhdan–Lusztig theory for affine Lie algebras at critical level

*Yale University, Email: gurbir.dhillon@yale.edu*

*Abstract:* A basic problem in representation theory is to calculate the characters of irreducible highest weight modules. For semisimple Lie algebras this was resolved by Beilinson–Bernstein and Brylinski–Kashiwara in the early 1980s. For affine Lie algebras, this was resolved by Kashiwara–Tanisaki at non-critical central charges in the early 1990s. At the remaining critical central charge certain cases are known by work of Arakawa–Fiebig, Feigin–Frenkel, Frenkel–Gaietsgory, Hayashi, and Malikov. In forthcoming joint work with David Yang, we calculate the irreducible characters for the regular block at critical level, confirming a conjecture of Feigin–Frenkel.

**Mee Seong Im** One-dimensional topological theories with zero-dimensional defects and finite state automata

*United States Naval Academy, Email: im@usna.edu*

*Abstract:* Quantum groups are related to 3-dimensional topological quantum field theories. Downsizing from three dimensions to one and from a ground field to a semiring, I will explain a surprising relation between topological theories for one-dimensional manifolds with defects and values in the Boolean semiring and finite-state automata and their generalizations. This is joint with Mikhail Khovanov.

**David Rose** Type C Webs

*University of North Carolina, Chapel Hill Email: davidrose@unc.edu*

*Abstract:* In his seminal 1996 paper, Kuperberg gives presentations for the categories of finite-dimensional representations of quantum groups associated to rank 2 simple complex Lie algebras (as braided pivotal categories). Such presentations underly various invariants in low-dimensional topology; in particular, they serve as a “foundation” for link homology theories. Kuperberg also poses the following problem: to find analogous descriptions of these categories for quantum groups associated with higher rank Lie algebras. In 2012, Cautis-Kamnitzer-Morrison solved this problem in type A using skew Howe duality, a technique that does not immediately extend to give a solution in other types.

In this talk, we will present a solution to Kuperberg’s problem in type C. Our proof combines results on pivotal categories and quantum group representations with diagrammatic/topological analogues of theorems concerning reduced expressions in the symmetric group. Time permitting, we’ll discuss some future directions. This work is joint with Elijah Bodish, Ben Elias, and Logan Tatham.

**Lev Rozansky** Quasi-quiver varieties for  $GL(M|N)$

*University of North Carolina, Chapel Hill Email: rozansky@math.unc.edu*

*Abstract:* There are well-known quiver varieties related to tensor products of the fundamental representations of  $GL(N)$  and to the corresponding link homology. I will show how to modify the quiver varieties in order to extend these constructions to the fundamental representations of  $GL(M|N)$ . The talk is based on a joint work with A. Oblomkov and R. Rimanyi.

**Andrew Manion** Decategorifying higher actions in Heegaard Floer homology

*North Carolina State Univeristy, Email: ajmanion@ncsu.edu*

*Abstract:* I will discuss the decategorification of the higher actions on bordered (sutured) Heegaard Floer strands algebras arising from joint work with Raphael Rouquier (and motivated by Douglas-Manolescu’s constructions in cornered Heegaard Floer homology). I will also discuss a new perspective on the sutured surfaces to which these algebras are assigned, namely the interpretation of these sutured surfaces as open-closed cobordisms, and try to explain a more flexible gluing theorem (related to open-closed TQFT) for the decategorifications of the algebras, recovering the decategorification of the cornered-Floer or higher-representation-theoretic gluing theorem as a special case.

**Cris Negron** Quantum  $SL(2)$  and logarithmic VOAs of type  $A_1$

*University of Southern California, Email: cnegron@usc.edu*

*Abstract:* I will discuss recent work with Terry Gannon, where we show that various non-semisimple tensor categories of representations for quantum  $SL(2)$  are identified with corresponding categories of modules for non-rational (aka logarithmic) vertex operator algebras. Such relationships were conjectured to exist in works of Bushlanov, Feigin, Gainutdinov, Semikatoev, and Tipunin from the mid 2000’s. At the conclusion I may touch on related, somewhat conjectural, “logarithmic TQFTs”

for quantum groups at roots of unity.

**Andrei Negut** On the trace of the affine Hecke category

*Massachusetts Institute of Technology, Email: anegut@mit.edu*

*Abstract:* We propose a connection between the horizontal trace of the affine Hecke category and the elliptic Hall algebra, mirroring known constructions for the finite Hecke category. Explicitly, we construct a family of generators of the affine Hecke category, compute certain categorified commutators between them, and show that their K-theoretic shadows match certain commutators in the elliptic Hall algebra. Joint work with Eugene Gorsky.

**Joshua Sussan** p-DG structures in higher representation theory

*City University of New York, Medgar Evers, Email: jsussan@mec.cuny.edu*

*Abstract:* One of the goals of the categorification program is to construct a homological invariant of 3-manifolds coming from the higher representation theory of quantum groups. The WRT 3-manifold invariant uses quantum groups at a root of unity. p-DG theory was introduced by Khovanov as a means to categorify objects at prime roots of unity. We will review this machinery and show how to construct categorifications of certain representations of quantum  $\mathfrak{sl}(2)$  at prime roots of unity.

**Ben Webster** Knot homology from coherent sheaves on Coulomb branches

*University of Waterloo, Email: ben.webster@uwaterloo.ca*

*Abstract:* Recent work of Aganagic details the construction of a homological knot invariant categorifying the Reshetikhin-Turaev invariants of minuscule representations of type ADE Lie algebras, using the geometry and physics of coherent sheaves on a space which one can alternately describe as a resolved slice in the affine Grassmannian, a space of G-monopoles with specified singularities, or as the Coulomb branch of the corresponding 3d quiver gauge theories. We give a construction of this invariant using an algebraic perspective on BFN's construction of the Coulomb branch, and in fact extend it to an invariant of annular knots. This depends on the theory of line operators in the corresponding quiver gauge theory and their relationship to non-commutative resolutions of these varieties (generalizing Bezrukavnikov's non-commutative Springer resolution).

## Titles and abstracts of contributed talks

**Rostislav Akhmechet:** Anchored foams and annular homology

*University of Virginia,, Email: ra5aq@virginia.edu*

*Abstract:* We discuss constructions of equivariant homology for links in the thickened annulus via closed foam evaluation. The thickened annulus is replaced by 3-space with a distinguished line. Foams are allowed to generically intersect the line and carry additional decorations at these intersection points. I will explain the construction in the  $\mathfrak{sl}(2)$  and  $\mathfrak{sl}(3)$  setting, based on joint work with Mikhail Khovanov, and mention work in preparation which extends the construction to  $\mathfrak{gl}(N)$  foams.

**Elijah Bodish** Semisimplification of  $\text{Tilt}(\mathfrak{sp}_{2n})$

*University of Oregon, Email: ebodish@uoregon.edu*

*Abstract:* I will describe a conjectural equivalence between the following two monoidal categories:

1. The semisimplification of the category of tilting modules for quantum  $\mathfrak{sp}_{2n}$  at certain small roots of unity.

2. The category of finite dimensional representations of the group  $O(2)$ .

I will explain how this conjecture would follow from a conjecture appearing in B.-Elias-Rose-Tatham's "Type C Webs". Given time I will try to explain how this equivalence appears to be related to Kazhdan-Lusztig cell theory and the geometry of nilpotent orbits.

**Mark Ebert** Derived super equivalences from odd categorified quantum group

*University of Southern California, Email: markeber@usc.edu*

*Abstract:* Since Chuang and Rouquier's pioneering work showing that categorical  $\mathfrak{sl}(2)$ -actions give rise to derived equivalences, the construction of derived equivalences has been one of the more prominent tools coming from higher representation theory. In this talk, we explain joint work with Aaron Lauda and Laurent Vera giving new super analogues of these derived equivalences stemming from the odd categorification of  $\mathfrak{sl}(2)$ . Just as Chuang and Rouquier used their equivalences to achieve new results on the modular representation theory of the symmetric group, we will discuss how our new super equivalences can be applied to the spin symmetric group.

**Tomas Gomez** A topological Rasmussen spectral sequence

*Johns Hopkins University, Email: tmejiag1@jhu.edu*

*Abstract:* There is a stable homotopy link invariant, called the symmetry breaking spectrum by Kitchloo, defined by geometrically lifting the Soergel bimodule construction of HOMFLY-PT homology. Under a suitable twisting of Borel equivariant cohomology, one may obtain a link invariant and a spectral sequence converging to it from HOMFLY-PT homology. This is a version of the Rasmussen spectral sequence, showing how this topological construction also captures  $\mathfrak{sl}(n)$  link homologies.

**Scott Larson** A categorification of the Lusztig-Vogan module

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*Abstract:* Continuous actions of real reductive groups are often studied by first linearizing the action to spaces related to functions, then using algebra via Lie algebras and compact groups (cf. Gelfand, Harish-Chandra, Vogan). This paradigm essentially simplifies to the easier problem of studying a complex algebraic group  $K$  acting on flag varieties.  $K$ -orbit closures are important for representation theory, are generalizations of Schubert varieties, and certain properties are explicitly determined via equivariant resolutions of singularities. In joint work with Anna Romanov, we provide a geometric and algebraic categorification of the Lusztig-Vogan module using the equivariant derived category. Our methods allow us to compute cohomology of all fibers of resolutions constructed quite generally, and generalize Soergel bimodule techniques from complex to real reductive groups.

**Cailan Li** The two color Ext Soergel calculus

*Columbia University, Email: ccl2166@columbia.edu*

*Abstract:* The monoidal category of Soergel Bimodules has a diagrammatic presentation by generators and relations. Replacing Hom spaces between Soergel Bimodules with the corresponding Ext groups, we obtain the monoidal category of Ext-enhanced Soergel Bimodules. We give a diagrammatic presentation for this monoidal category in the case of a dihedral group.

**Matthew McMillan** A tensor 2-product for 2-representations of  $\mathfrak{sl}(2)$

*University of California, Los Angeles, Email: matthew.mcmillan@math.ucla.edu*

*Abstract:* The categorification program for TQFTs has long sought a braided monoidal structure for the 2-category of 2-representations of Kac-Moody 2-algebras. Such structure requires a general construction for the tensor 2-product of 2-reps. Webster has given a diagrammatic categorification for products of simple 2-reps, and with Losev defined some axioms that determine these uniquely. Rouquier has formulated a general construction, but it returns  $\mathcal{A}_\infty$ -categories as the products of 2-reps given by (dg)-algebras. Manion-Rouquier applied this construction in the case of  $\mathfrak{gl}(1|1)$  where homotopical complications disappear. We present a general construction preserving the 2-category of algebras for the case of  $\mathfrak{sl}_2$ , specifically the product of the fundamental rep  $\mathcal{L}(1)$  and an arbitrary rep. We study the output for  $\mathcal{L}(1)$  times  $\mathcal{L}(n)$  and compare with the known categorification in this case.

**Eric Samperton** Quantum computation and link homology

*University of Illinois at Urbana-Champaign, Email: smprtn@illinois.edu*

*Abstract:* Topological quantum computation is a much-studied approach to building real-world quantum computers based on the physics of topological phases and the mathematics of (2+1)-dimensional unitary TQFTs such as the Jones polynomial. It is natural to wonder if there is a connection between quantum computation and categorified link homology invariants such as Khovanov homology. I'll introduce two preliminary answers, depending on whether one takes their coefficients with positive characteristic or with zero characteristic. The former is related to quantum error correcting codes. The latter is related to quantum phase estimation and QMA (Quantum

Merlin Arthur), the quantum analog of the complexity class NP.

**Arik Wilbert** Real Springer fibers and odd arc algebras

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*Abstract:* Arc algebras were introduced by Khovanov in a successful attempt to lift the quantum  $sl_2$  Reshetikhin-Turaev invariant for tangles to a homological invariant. When restricted to knots and links, Khovanov's homology theory categorifies the Jones polynomial. Osvath-Rasmussen—Szabo discovered a different categorification of the Jones polynomial called odd Khovanov homology. Recently, Naisse-Putyra were able to extend odd Khovanov homology to tangles using so-called odd arc algebras which were originally constructed by Naisse-Vaz. The goal of this talk is to discuss a geometric approach to understanding odd arc algebras and odd Khovanov homology using Springer fibers over the real numbers.

**Yaolong Shen** The  $q$ -Brauer algebra and  $i$ -Schur duality

*University of Virginia, Email: ys8pfr@virginia.edu*

*Abstract:* Brauer introduced the Brauer algebra, and established the double centralizer property between it and the orthogonal group or symplectic group. Wenzl defined a  $q$ -deformation of the Brauer algebra which contains the type A Hecke algebra as a natural subalgebra. It is well known that Jimbo-Schur duality relates Hecke algebras and quantum groups of type A.

In recent years, Bao and Wang have formulated a  $q$ -Schur duality between a type B Hecke algebra and an  $i$ -quantum group arising from quantum symmetric pairs. In this talk we focus on  $i$ -quantum groups which specialize to the orthogonal or symplectic Lie algebra at  $q=1$  and introduce an  $i$ -Schur duality between them and the  $q$ -Brauer algebra. We also develop a natural bar involution and construct a Kazhdan-Lusztig type canonical basis on the  $q$ -Brauer algebra. This is joint work with Weideng Cui.

**Melissa Zhang** Constructions toward topological applications of  $U(1) \times U(1)$ -equivariant Khovanov homology

*University of Georgia, Email: melissa.li.zhang@gmail.com*

*Abstract:* In 2018, Khovanov and Robert introduced a version of Khovanov homology over a larger ground ring, termed  $U(1) \times U(1)$ -equivariant Khovanov homology. This theory was also studied extensively by Taketo Sano. Ross Akhmechet was able to construct an equivariant annular Khovanov homology theory using the  $U(1) \times U(1)$ -equivariant theory, while the existence of a  $U(2)$ -equivariant annular construction is still unclear.

Observing that the  $U(1) \times U(1)$  complex admits two symmetric algebraic gradings, those familiar with knot Floer homology over the ring  $F[U, V]$  may naturally ask if these filtrations allow for algebraic constructions already seen in the knot Floer context, such as Ozsváth-Stipsicz-Szabó's Upsilon. In this talk, I will describe the construction and properties of such an invariant. I will also discuss some ideas on how future research might use the  $U(1) \times U(1)$  framework to identify invariants similar to those constructed from knot Floer homology over  $F[U, V]$ , and speculate on the topological information these constructions might illuminate.

This is based on joint work with Ross Akhmechet.

**Weinan Zhang** Braid group symmetries on i-quantum groups

*University of Virginia, Email: wz3nz@virginia.edu*

*Abstract:* Introduced by Lusztig in the 1990s, the braid group symmetries constitute an essential part in the theory of quantum groups. The i-quantum groups are coideal subalgebras of quantum groups, which arises from quantum symmetric pairs. In the recent years, many results for quantum groups have been generalized to i-quantum groups.

In this talk, I will present our construction of relative braid group symmetries, associated to the relative Weyl group of a symmetric pair, on i-quantum groups of arbitrary finite types. These new symmetries inherit most properties of Lusztig's symmetries, including compatible relative braid group actions on modules. This is joint work with Weiqiang Wang.