Workshop on Representation Theory, Combinatorics, and Geometry

October 19-21, Charlottesville, Virginia

Friday, Oct 19 in Wilson 301

1:30-1:35 - Opening remarks

1:35-2:20 Anne Schilling (UC Davis) - Characterization of queer supercrystals

> We provide a characterization of the crystal bases for the quantum queer superalgebra recently introduced by Grantcharov et al. This characterization is a combination of local queer axioms generalizing Stembridge's local axioms for crystal bases for simply-laced root systems, which were recently introduced by Assaf and Oguz, with further axioms and a new graph G characterizing the relations of the type A components of the queer crystal. We provide a counterexample to Assaf's and Oguz' conjecture that the local queer axioms uniquely characterize the queer supercrystal. We obtain a combinatorial description of the graph G on the type A components by providing explicit combinatorial rules for the odd queer operators on certain highest weight elements. This is joint work with Maria Gillespie, Graham Hawkes, and Wencin Poh.

2:20-2:50 coffee

2:50-4:00 Participant talks

1. **Mandy Welch (Virginia Tech)** - Double Affine Bruhat Order

   > Given a finite Weyl group W_{fin} with root system Φ_{fin}, one can create the affine Weyl group W_{aff} by taking the semidirect product of the translation group associated to $Q^\vee$, the coroot lattice for Φ_{fin}, with W_{fin}. The double affine Weyl semigroup W can be created by using a similar semidirect product where one replaces W_{fin} with W_{aff} and $Q^\vee$ with the Tits cone of W_{aff}. We classify cocovers and covers of a given element of W with respect to the Bruhat order, specifically when W is associated to a finite root system that is irreducible and simply laced. We show two approaches: one adapting the work of Lam and Shimozono, and its strengthening by Milicevic, where cocovers are characterized in the affine case using the quantum Bruhat graph of W_{fin}, and another, which takes a more geometrical approach by using the length difference set defined by Muthiah and Orr.

2. **Chun-Ju Lai (University of Georgia)** - Schur algebras and quantum symmetric pairs with unequal parameters

   > It has been shown in a recent work by Bao-Wang-Watanabe that the multiparameter Schur algebras of type B/C admit a master Schur duality that recovers one-parameter Schur duality of type B/C/D by specialization. Motivated by their work, we establish some favorable properties for the multiparameter Schur algebras using Lusztig's bar-invariant basis for Hecke algebras with unequal parameters. We further establish a two-parameter upgrade of quantum

> symmetric pairs of type A III/IV in the sense of Beilinson-Lusztig-MacPherson. We obtain, at the specialization associated to any weight function, the canonical basis of the Schur algebras and the coideal subalgebras.

3. **Joshua Kiers (UNC Chapel Hill)** - Recent progress on the saturation conjecture for type D

> The decomposition of a tensor product of Lie group representations into irreducibles is a standard problem in representation theory. The question is controlled by a certain semigroup, the tensor cone. A related semigroup, the saturated tensor cone, can be more easily studied thanks to geometric invariant theory. In this talk we discuss known results on the saturation conjecture: when are these two cones equal? We list recent results for the even spin groups. Along the way, we introduce and use formulas for the extremal rays of the saturated cone.

4:15-5:00 Ben Webster (University of Waterloo) - Coherent sheaves on Hilbert schemes through the Coulomb lens

> The Hilbert scheme of the plane (or more generally, the resolution of the type A Kleinian singularity) has a surprising description as a Coulomb branch in the sense of Braverman, Finkelberg and Nakajima; this gives a rather odd looking presentation of the projective coordinate ring of this variety. It turns out that this presentation is very well adapted to describing the tilting generators constructed using Bezrukavnikov and Kaledin's perspective; of course, there are many other interesting aspects of the combinatorics of coherent sheaves on these varieties, which hopefully the audience will have some thoughts about understanding this way.

5:15-6:00 Voula Collins (University of Connecticut) - Cohomology of the cotangent bundle to a Grassmannian and puzzles

> Maulik and Okounkov described a stable basis for the T-equivariant cohomology ring $H^*_{T\times\mathbb{C}^\times}(T^*Gr_k(\mathbb{C}^n))$ of the cotangent bundle to a Grassmannian. A natural idea to consider is the product structure of these basis elements. In this talk I will give a way to compute the structure constants for the projective case, as well as a conjectural positive formula using puzzles. I will also discuss ideas for modifying this formula to apply to a general Grassmannian using R-matrices, based on work done by Knutson and Zinn-Justin.

Saturday, Oct 20 in Wilson 301

9:00-9:45 Jinkui Wan (Beijing Institute of Technology, visiting University of Virginia) - Stability of the centers of group algebras of $GL_n(q)$

> The center of the integral group algebra of the general linear group $GL_n(q)$ over a finite field admits a filtration with respect to the reflection length. We show that the structure constants of the associated graded algebras are independent of n, and this stability leads to a universal stable center with positive integer structure constants which governs these graded algebras for all n. Various structure constants of the stable center are computed and several conjectures are formulated. Analogous stability properties for symmetric groups and wreath products were established earlier by Farahat-Higman and Wang. This is a joint work with Weiqiang Wang.

9:45-10:30 coffee

10:30-11:40 Participant talks

1. Michael Reeks (University of Ottawa) - Traces of tensor product categories

   > The trace is a decategorification functor which can often reveal additional structure not visible in the Grothendieck group. For instance, the categories of modules over the cyclotomic KLR algebra associated to a Lie algebra $\mathfrak{g}$ of type ADE have Grothendieck groups isomorphic to highest weight integrable representations of the quantum group $U_q(\mathfrak{g})$, while their traces are isomorphic to Weyl modules over the current algebra of $\mathfrak{g}$. Webster introduced a generalization of cyclotomic KLR algebras called tensor product algebras. Modules over the these algebras categorify tensor products of highest weight integrable modules of $U_q(\mathfrak{g})$. In this talk, we investigate the trace of Webster's tensor product categorification, and show that it is isomorphic to a tensor product of Weyl modules. This is joint work with Christopher Leonard (Virginia).

2. **Colleen Robichaux (University of Illinois, Urbana-Champaign)** - Vanishing of Littlewood-Richardson polynomials is in P

   > J. DeLoera-T. McAllister and K. D. Mulmuley-H. Narayanan-M. Sohoni independently proved that determining the vanishing of Littlewood-Richardson coefficients has strongly polynomial time computational complexity. Viewing these as Schubert calculus numbers, we prove the generalization to the Littlewood-Richardson polynomials that control equivariant cohomology of Grassmannians. We construct a polytope using the edge-labeled tableau rule of H. Thomas-A. Yong. Our proof then combines a saturation theorem of D. Anderson-E. Richmond-A. Yong, a reading order independence property, and E. Tardos' algorithm for combinatorial linear programming. This is joint work with A. Adve and A. Yong.

3. **Balazs Elek (University of Toronto)** - Kirillov-Reshetikhin crystals and Cacti

   > Kirillov-Reshetikhin modules are certain finite-dimensional representations of loop groups. They have been conjectured to have crystal bases, in the sense of Kashiwara, and this has been proven in most Cartan types, but with a case by case argument. We will discuss some work in progress that could lead to a more unified description of the affine crystal structure on classically irreducible Kirillov-Reshetikhin crystals using the cactus group action studied by Henriques-Kamnitzer and Halacheva.

12-12:45 Alex Yong (University of Illinois at Urbana-Champaign) - Complexity, Combinatorics, and Newton polytopes

> The Nonvanishing decision problem asks if a coefficient of a polynomial is nonzero. Work of C. Monical, N. Tokcan and the speaker explicated that many polynomials in algebraic combinatorics admit combinatorial counting rules and enjoy having "saturated Newton polytopes". Therefore, in amenable cases, Nonvanishing is in the complexity class of problems with "good characterizations". This suggests a new algebraic combinatorics paradigm for complexity theory. The talk will focus on the case of Schubert polynomials, a basis of all polynomials. These appear in the study of cohomology rings of flag varieties. We give a handy tableau criterion for Nonvanishing, from which we deduce the first polynomial time algorithm. This is based on joint work with Anshul Adve and Colleen Robichaux.

12:45-2:30 lunch break

2:30-3:15 Eric Carlsson (UC Davis) - Affine Schubert calculus and diagonal coinvariants

> I'll present some new results with Alexei Oblomkov that 1. identify the diagonal coinvariant algebra DRn with the homology of the (ramified) (n,n+1)-affine springer fiber using affine Schubert calculus. This is compatible with the constructions of Oblomkov-Yun, but is a more elementary construction. 2. This leads to an elementary filtration of the DRn, with an algebraic presentation of the subquotients, and 3. We produce an explicit monomial basis, categorifying the Haglund-Loehr formula, for the Hilbert series of DRn. This realizes some ideas of Hikita and others, who realized, among other things, that the fixed points of this Springer fiber were in bijection with parking functions, and expected that there should be a proof of the Haglund-Loehr formula, or more general shuffle theorem along these lines.

3:15-4:00 coffee

4:00-4:45 Anna Pun (Drexel University) - Catalan Functions and k-Schur functions

> Li-Chung Chen and Mark Haiman studied a family of symmetric functions called Catalan (symmetric) functions which are indexed by pairs consisting of a partition contained in the staircase (n-1, ..., 1,0) (of which there are Catalan many) and a composition weight of length n. They include the Schur functions ,the Hall-Littlewood polynomials and their parabolic generalizations. They can be defined by a Demazure-operator formula, and are equal to GL-equivariant Euler characteristics of vector bundles on the flag variety by the Borel-Weil-Bott theorem. We have discovered various properties of Catalan functions, providing a new insight on the existing theorems and conjectures inspired by Macdonald positivity conjecture. A key discovery in our work is an elegant set of ideals of roots that the associated Catalan functions are k-Schur functions and proved that graded k-Schur functions are G-equivariant Euler characteristics of vector bundles on the flag variety, settling a conjecture of Chen-Haiman. We exposed a new shift invariance property of the graded k-Schur functions and resolved the Schur positivity and k-branching conjectures by providing direct combinatorial formulas using strong marked tableaux. We conjectured that Catalan functions with a partition weight are k-Schur positive which strengthens the Schur positivity of Catalan function conjecture by Chen-Haiman and resolved the conjecture with positive combinatorial formulas in cases which capture and refine a variety of problems. This is joint work with Jonah Blasiak, Jennifer Morse and Daniel Summers
> arXiv1804.03701

5:00-5:45 Yiqiang Li (SUNY at Buffalo) - Quiver varieties and symmetric pairs

> To a simply-laced Dynkin diagram, one can attach a complex simple Lie algebra, say $\mathfrak{g}$, and a class of Nakajima (quiver) varieties. The latter provides a natural home for a geometric representation theory of the former. If the algebra $\mathfrak{g}$ is further equipped with an involution, it leads to a so-called symmetric pair $(\mathfrak{g}, \mathfrak{k})$, where $\mathfrak{k}$ is the fixed-point subalgebra under involution. In this talk, I'll present a recent study of fixed-point loci of Nakajima varieties under certain involutions and provide bridges at several levels between symmetric pairs and Nakajima varieties.

Sunday, Oct 21 in Wilson 301

9:00-9:45 Eugene Gorsky (UC Davis) - Khovanov-Rozansky homology and Hilbert schemes of points

> Khovanov and Rozansky introduced a link homology theory which categorifies the HOMFLY polynomial. This invariant has a lot of interesting properties, but it is notoriously hard to compute. I will discuss recent progress in understanding HOMFLY homology and its conjectural relation to algebraic geometry of the Hilbert scheme of points on the plane. The talk is based on joint works with Matt Hogancamp, Andrei Negut and Jacob Rasmussen.

9:45-10:30 coffee

10:30-11:40 Participant talks

1. **Molly Lynch (North Carolina State University)** - Relations in doubly laced crystal graphs via discrete Morse theory

   > Many crystals have a natural poset structure. In this talk, we explore the combinatorics of crystal posets associated to highest weight representations of finite Cartan type. To do so, we use a tool from topological combinatorics called lexicographic discrete Morse functions. This allows us to relate the Möbius function of an interval in a crystal poset to the types of relations that exist among crystal operators in that interval. Additionally, we show that crystal posets associated to highest weight representations of types B_2 and C_2 are not lattices.

2. **Henry Kvinge (Colorado State University)** - Heisenberg categories, towers of algebras, and up/down transition functions

   > Deep results have been obtained at the intersection of representation theory, probability theory, and combinatorics via the study of the asymptotic properties of towers of algebras and their associated representations. A basic component in many of these constructions is the concept of a coherent measure M_n on certain representations of A_n along with associated up/down transition functions that allow one to relate M_n to a probability measure M_{n-1} on representations of A_{n-1} and a probability measure M_{n+1} on representations of A_{n+1}. It has been shown in many cases that these transition functions can be encoded as families of symmetric functions and this has been useful in understanding their properties. Surprisingly, these same symmetric functions appear very naturally within certain recent categorifications of the Heisenberg algebra also using towers of algebras. In this talk we will review the appearance of transition functions in the center of Heisenberg categories and describe how these results suggest a way to generalize the coherent measure/transition function framework to a broader class of (potentially non-semisimple) towers of Frobenius extensions.

3. **Wai Ling Yee (University of Windsor)** - The Unitary Dual Problem and Hall-Littlewood Polynomials

   > The Unitary Dual Problem is one of the most important open problems in mathematics: classify the irreducible unitary representations of a group. The standard approach is to study representations admitting invariant Hermitian forms, compute the signatures of those Hermitian forms, and determine which forms are definite. Signatures of invariant Hermitian forms on Verma modules may be computed by a philosophy of Vogan: deform the highest weight and keep track of how the signature changes as one crosses reducibility hyperplanes. Due to recursion, the formula for the signature character is complicated, involving products of

> signs associated to crossing reducibility hyperplanes and powers of 2. In recent work, it is shown that all of the complexity can be encoded by the affine Hecke algebra: the signature character of an invariant Hermitian form on a Verma module is a Hall-Littlewood polynomial summand evaluated at q=-1 times a version of the Weyl denominator.

12-12:45 Nicolas Guay (University of Alberta) - Twisted Yangians for symmetric pairs of types B, C and D

> I will introduce twisted Yangians associated to symmetric pairs of types B, C and D which are similar to those of type A introduced by G. Olshanski around 1990 and which have been quite well studied. Twisted Yangians are families of co-ideal subalgebras of Yangians with an interesting representation theory and applications to theoretical physics. After a discussion of some of their properties, I will present classification results for their irreducible finite-dimensional modules. This is joint work with Vidas Regelskis and Curtis Wendlandt.

12:45-2:30 lunch break

2:30-3:15 Richard Rimanyi (UNC Chapel Hill) - Motivic characteristic classes, Hall algebras, and Donaldson-Thomas type identities

> The equivariant Chern-Schwartz-MacPherson (CSM) class and the equivariant Motivic Chern (MC) class are fine characteristic classes of singular varieties in cohomology and K theory—and their theory overlaps with the theory of Okounkov's stable envelopes. We study CSM and MC classes for the orbits of Dynkin quiver representations. We show that the problem of computing the CSM and MC classes of all these orbits can be reduced to some basic classes c^o_β, C^o_β parameterized by positive roots β. We prove an identity in a deformed version of Kontsevich-Soibelman's Cohomological (and K-theoretic) Hall Algebra (CoHA, KHA), namely, that a product of exponentials of c^o_β, C^o_β classes formally depending on a stability function Z, does not depend on Z. This identity—which encodes infinitely many identities among rational functions in infinitely many variables—has the structure of Donaldson-Thomas type quantum dilogarithm identities. Using a wall-crossing argument we present the c^o_β, C^o_β classes as certain commutators in the CoHA, KHA.

3:15-4:00 coffee

4:00-4:45 Daniel Orr (Virginia Tech) - Semi-infinite flag manifolds and the nonsymmetric q-Toda system

> The graded characters of Demazure submodules of level-zero extremal weight modules over a simply-laced quantum affine algebra are solutions to certain difference equations known as the nonsymmetric q-Toda system (arising from nil-DAHA). By results of Kato and Kato-Naito-Sagaki, these graded characters ''control'' the equivariant K-theory of the associated semi-infinite flag manifold. I will explain some cases in which it has been possible to convert the difference equations into meaningful statements in equivariant K-theory. This is based on joint works with Feigin and Makedonskyi and with Naito and Sagaki.